2008

Mathematics Curriculum Guide

Catholic Diocese of Wilmington, Delaware

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ACKNOWLEDGEMENTS

The Catholic Schools Office of the Diocese of Wilmington would like to acknowledge the effort and dedication of the following individuals who volunteered their time over a two year period to revise and publish the Diocesan Mathematics Curriculum Guidelines.

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Mission

The Catholic school has the responsibility to prepare all students to function effectively in today's society and to bring Christian values to their world. Integral to the complete formation of the child in our Catholic schools is the study of Mathematics. Students of the twenty-first century must be taught to value Mathematics and become competent and confident in reasoning, making connections, and communicating in order to be better problem solvers. They should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibility.

Vision

As life-long learners, we are challenged to use God's gifts to better understand and improve the world around us. We recognize that we live in a world that is increasingly mathematical and technological and that our students' futures depend on their mathematical competency. Students should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibly. Teaching strategies and learning experiences must be varied, meaningful, and engaging to students.

Philosophy

Mathematics is learned through an approach that begins with concrete explorations and leads students to an understanding of symbolic representations. All students must have equal access to rigorous, high quality instruction to become mathematically literate. The uniqueness of each student should be nurtured by using differentiated strategies in response to various learning styles. A broad variety of assessments must provide multiple indicators of student achievement.

Communicating mathematically enables students to solve problems by acquiring information through reading, listening, and observing. Students will be able to translate information into mathematical language and symbols, process the information mathematically, and present the results in written, oral, and visual formats to demonstrate their mathematical literacy.

Students achieve mastery of computational skills through the employment of age-appropriate materials while also developing higher-level critical thinking skills. In our progressively changing world, students need to know how to properly utilize innovative tools, media, and technology to solve cross-curricular mathematical problems. Technology, however, is not a replacement for the comprehension of mathematical concepts.

The Mathematics program prepares students to fulfill personal ambitions and career goals in an ever changing world. Classrooms that encourage investigation, collaboration, and

resourcefulness in the problem solving process empower students beyond the classroom. It is through the cornerstones of communication, teamwork, and opportunity that we instill into our students a deeper appreciation and knowledge of mathematics so that they may become productive Catholic citizens of the world.

Goals

All students will:

1. Learn to appreciate mathematics, reason mathematically, and communicate mathematically.
2. Utilize their mathematical skills to become competent problem solvers.
3. Make mathematical connections to real life situations and to other areas of the curriculum.
4. Use technology appropriately and effectively.
5. Apply ethical and critical thinking.

Expectations for Learning

We commit to the following expectations:

1. That all grade levels students:
    - Learn to think critically, logically, ethically, and analytically
    - Learn to express ideas orally and in writing using correct mathematical terminology
    - Learn to apply the techniques of mathematics to real world situations
    - Understand that mathematics is important to function in today's world
    - Utilize technology responsibly
1. That computers, calculators, manipulatives and other tools of learning should be used routinely as an integral part of both instruction and assessment.
2. That mathematics teachers be encouraged to participate in professional development activities.
3. That mathematics coordinators hold regularly scheduled faculty meetings to facilitate communication and to analyze the strengths and weaknesses within the program.
4. That the teacher utilize the mathematics curriculum guidelines for grade level instruction.
5. That teachers provide differentiated instruction and assessment.

National Council of Teachers of Mathematics (NCTM)

Principles and Standards for School Mathematics

The six **NCTM Principles** describe particular and important features of high-quality mathematics education. The ten **NCTM Standards** describe the mathematical *content* and *processes* that students should learn. Together, the Principles and Standards constitute a vision to guide educators as they strive for the continual improvement of mathematics education in classrooms, schools, and educational systems.

Principles (NCTM)

The Equity Principle

Excellence in mathematics education requires equity – high expectations and strong support for all students.

All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study – and support to learn – mathematics. This does not mean that all students should be treated the same. But all students need access each year they are in school to a coherent, challenging mathematics curriculum that is taught by competent and well supported teachers. Equity does not imply lowering expectations for any group of students. Rather expectations must be raised for all – mathematics can and must be learned by all students. There is no conflict between equity and excellence.

The Curriculum Principle

A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.

School mathematics curricula should focus on mathematics content and processes that are worth the time and attention of students. Mathematics topics can be considered important for different reasons, such as their utility in developing other mathematical ideas, in linking different areas of mathematics, or in deepening students' appreciation of mathematics as a discipline. A coherent curriculum allows students to effectively organize and integrate important mathematical ideas.

The Teaching Principle

Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn.

Teachers have different styles and strategies for helping students learn particular mathematical ideas, and there is no one "right way" to teach. However, regardless of style, teachers must decide what aspects of a task to highlight, how to organize and orchestrate the work of the students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them. Selecting and using suitable curricular material, using appropriate instructional tools and techniques, and engaging in reflective practice and continuous self-improvement are actions good teachers take every day. A good curriculum is not sufficient for effective learning and teaching. Teachers are required each day to make choices about how the learning environment will be structured and what mathematics will be emphasized. These decisions determine, to a large extent, what students learn. Effective teaching conveys the belief that each student can understand mathematics and that each will be supported in accomplishing this goal.

The Learning Principle

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

Learning with understanding is essential to enable students to solve the new kinds of problems they will inevitably face in the future. Unfortunately, learning without understanding has long been a common outcome of school mathematics. Students who memorize facts and procedures without understanding often are not sure when or how to use what they know and such learning is often quite fragile. Mathematics makes more sense and is easier to remember and to apply when students connect new knowledge to existing knowledge in meaningful ways. Learning with understanding also makes subsequent learning easier. Students' understanding of mathematical ideas can be built throughout their school years if they actively engage in tasks and experiences designed to deepen and connect their knowledge. Learning with understanding can be further enhanced by classroom interactions, as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills.

The Assessment Principle

A*ssessment should support the learning of important mathematics and furnish useful information to both teachers and students.*

Assessment should be more than merely a test at the end of instruction to gauge learning. It should be an integral part of instruction that guides teachers and enhances students' learning. Teachers should be continually gathering information about their students through questions, interviews, writing tasks, and other means. They can then make appropriate decisions about such matters as reviewing materials, re-teaching a difficult concept, or providing something

more or different for students who are struggling or need enrichment. To be consistent with the Learning Principle, assessments should focus on understanding as well as procedural skills. Because different students show what they know and can do in different ways, assessments should also be done in multiple ways, such as open-ended questions, constructed-response task, selected-response items, performance tasks, observations, conversations, journals, and portfolios. Constructed-response or performance-tasks may better measure a student's capacity to apply mathematics in complex or new situations.

The Technology Principle

Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

Calculators and computers are reshaping the mathematics landscape, and school mathematics should reflect those changes. Students can learn more mathematics more deeply with the appropriate and responsible use of technology. They can make and test conjectures. They can work at higher levels of generalization or abstraction. Every student should have access to appropriate technology to facilitate his or her mathematics learning. Technology also offers options for students with special needs. Some students may benefit from the more constrained and engaging task situations possible with computers. Students with physical challenges can become much more engaged in mathematics using special technologies. Technology cannot replace the mathematics teacher, nor can it be used as a replacement for basic understandings and intuitions. The teacher must make prudent decisions about when and how to use technology and should ensure that the technology is enhancing students' mathematical thinking.

Standards for School Mathematics

The **Standards** describe a connected body of mathematical understandings and competencies, and are a comprehensive foundation recommended for all students. They describe the mathematical understanding, knowledge, and skills that students should acquire from kindergarten through grade 12.

The Content Standards – *Number and Operations, Algebra, Geometry, Measurement, Data Analysis and Probability* – reflect the important mathematical content that should serve as the basis of the Diocese of Wilmington mathematics program, and explicitly describe the content students should learn. The five Process Standards – *Problem Solving, Communication, Reasoning and Proof, Connections, and Representation* – highlight important and crucial ways of acquiring and using content knowledge.

The Process Standards are not isolated components of the curriculum, but should be used in the study of the Content Standards. Students will attain, and be assessed on, the process standards while pursuing the various content standards. Problem solving is a central process

standard and students will learn many process standards within the context of the problem-solving standard.

Content Standards

Number and Operations

Number pervades all areas of mathematics. The other four Content Standards as well as all five Process Standards are grounded in understanding number. Central to this standard is the development of number sense, which allows students to naturally combine or decompose numbers, solve problems using the relationships among operations and knowledge of the base-ten system, and make a reasonable estimate for the answer to a problem. Computational fluency – having and using efficient and accurate methods for computing – is essential. Students should be able to perform computations in different ways, including mental calculations, estimation, and paper-and-pencil calculations using mathematically sound algorithms. All students should use calculators at appropriate times, setting the calculator aside when the instructional focus is on developing computational algorithms.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- understand numbers, ways of representing numbers, relationships among numbers, and number systems;
- understand meanings of operations and how they relate to one another;
- compute fluently and make reasonable estimates.

Algebra

The ideas of algebra are a major component of the school mathematics curriculum and help to unify it. Mathematical investigations and discussions of arithmetic and its properties frequently include aspects of algebraic reasoning. Such experiences present rich contexts and opportunities for enhancing mathematical understanding and are an important precursor to the more formalized study of algebra in the middle and secondary grades. A strong foundation in algebra should be in place by the end of the eighth grade, and all high school students should pursue ambitious goals in algebra.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

Geometry

Geometry and spatial sense are fundamental components of mathematics learning. They offer ways to interpret and reflect on our physical environment and can serve as tools for the study of other topics in mathematics and science. Geometry is a natural area of mathematics for the development of students' reasoning and justification skills that build across the grades.

Geometry should be learned using concrete models, drawings, and dynamic software. As the study of the relationships among shapes and their properties becomes more abstract, students should come to understand the role of definitions and theorems and be able to construct their own proofs.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement

The study of measurement is crucial in the K-12 mathematics curriculum because of its practicality and pervasiveness in many aspects of everyday life. Measurement is possibly the area of mathematics that is most important when considering everyday applications of mathematics, and highlights connections between mathematics and areas outside of the school curriculum such as social studies, science, art, and physical education. The study of measurement helps students establish connections within mathematics and provides an opportunity for learning about and unifying ideas concerning number and operations, algebra, geometry, statistics, probability, and data analysis.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

Data Analysis and Probability

To analyze data and reason statistically are essential to be an informed citizen, employee, and consumer. The amount of statistical information available to help make decisions in business, politics, research, and everyday life is staggering. Through experiences with the collection and analysis of data, students can learn to make sense of and interpret information and allow them to make appropriate arguments and recognize inappropriate arguments as well.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

Process Standards

Problem Solving

Problem solving means engaging in a task for which the solution process is not known in advance. Good problem solvers have developed a "mathematical disposition" which allows them to analyze situations in mathematical terms. They have developed a range of strategies for developing a solution to a problem, have learned to monitor and adjust the strategies they choose to use in the process of solving a specific problem, and can compare and contrast solutions and problems.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Communication

As students are asked to communicate orally or in writing about the mathematics they are studying, they gain insights into their own thinking. In order to communicate their thinking to others, they naturally reflect on their learning and organize and consolidate their thinking about mathematics. Students should be encouraged and expected to increase their ability to express themselves clearly and coherently over time. In particular, the ability to express thoughts and describe solutions in writing should be a major focus of the mathematics curriculum.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking clearly and coherently to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Reasoning and Proof

Systematic reasoning is a defining feature of mathematics. Exploring, justifying, and using mathematical conjectures are common to all content areas and, with different levels of rigor, all grade levels. By the end of secondary school, students should be able to understand and produce some mathematical proofs – logically rigorous deductions of conclusions from mathematical hypotheses – and should appreciate the value of such arguments.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;

- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and proof.

Connections

Mathematics is an integrated field of study, even though it is often studied in separate areas or topics. Viewing mathematics as a whole helps students learn that mathematics is not a set of isolated skills and arbitrary rules. Focusing on mathematics in context and establishing mathematical connections makes it easier to apply mathematical knowledge and makes it less likely that students will forget or misapply important mathematical skills and rules.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

Representation

Representations are necessary to students' understanding of mathematical concepts and relationships. They allow students to communicate mathematical approaches, arguments, and understandings to themselves and others. Appropriate representations allow students to recognize connections among related concepts, and lead to efficient methods of solving problems.

It is important to encourage students to represent their mathematical ideas in ways that make sense to them, even if those representations are not conventional. At the same time, students should learn conventional forms of representation in ways that facilitate their learning of mathematics and their communication with others about mathematical ideas.

Instructional programs from kindergarten through grade 12 should enable **all** students to:

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

IMPLEMENTATION OF A MIDDLE SCHOOL ACCELERATED MATHEMATICS PROGRAM

***As life-long learners**, we are challenged to use God's gifts to better understand and improve the world around us. We recognize that we live in a world that is increasingly mathematical and technological and that our students' futures depend on their mathematical competency. Students should be able to assimilate new information, solve unfamiliar problems in unconventional ways, and work cooperatively as well as independently. They should also be able to interpret issues, think critically and ethically, and act responsibly. Teaching strategies and learning experiences must be varied, meaningful, and engaging to students.*

All elementary schools in the Diocese of Wilmington are encouraged to implement a middle school accelerated mathematics program. The program should begin in the sixth grade and should include only those students entering these grades who meet the criteria outlined in these guidelines of implementation.

An accelerated sixth, seventh, and eighth grade curricula shall include skills and concepts which are presented in the standards for grades sixth, seventh, and eighth. Pre-Algebra concepts are incorporated into the standards for these grades.

These courses should not replace the current mathematics courses offered in the middle school classes in the diocese. If appropriate resources are available schools are encouraged to provide a full and rigorous Algebra I program.

It is assumed that students in an Algebra I program in the eighth grade will be well-prepared to take the Algebra placement test for high school. It shall not be assumed that placement in an Algebra I program guarantees placement in Geometry in the ninth grade, as each high school has its own set of guidelines for admittance into Geometry in the ninth grade.

MIDDLE SCHOOL ACCELERATED MATHEMATICS PROGRAM ADMISSION CRITERIA

It is the hope of the Mathematics Curriculum Revision Committee that the following criteria be implemented in all elementary schools throughout the Diocese.

A student shall be placed in the accelerated mathematics program if he/she has:

- Achieved a composite mathematics score on the Terra Nova in the 85th percentile or higher in the year previous to admission into the program.

- Achieved a minimum score of 85% on their final report card at the conclusion of the year previous to admission into the program.

- For Algebra: Achieved a minimum score of 85% on an Algebra Readiness Test at the conclusion of seventh grade. New students will need to be evaluated.

- Received a recommendation from previous mathematics teachers who believe the student has demonstrated an appropriate maturity level, good work habits, and fundamental problem solving skills. In all, the student must be an aggressive learner (completes all homework, asks questions to clarify, good study habits, etc.)

- Admittance into program one year does not ensure participation the following year.

- Each student will be evaluated again during the first mid-trimester, at the end of the first trimester, and at the end of the current school year. Replacement in the normal math classes should be considered if a student has fallen far behind.

- This does not ensure that the student should be taking Algebra 1 during their freshman year.